1. Definition of some "global" cardinal functions. Elementary inequalities. The zero-dimensional case.

By a space we mean a Tychonoff topological space (even if many statements will be true under weaker assumptions.) All cardinal functions defined below are assumed to be infinite; thus, if in some trivial case a function of X defined as below turns out to be finite, then multiply it by ω.

The *density* of X is $d(X) = \min\{\tau : X$ has a dense subspace of cardinality $\leq \tau\}$. If $d(X) = \omega$ then X is called *separable*.

The *weight* of X is $w(X) = \min\{\tau : X$ has a base of cardinality $\leq \tau\}$. If $w(X) = \omega$ then X is called *second countable* (this is equivalent to being separable and metrizable.)

The *Lindelöf number* of X is $l(X) = \min\{\tau :$ every open cover of X contains a subcover of cardinality $\leq \tau\}$. If $l(X) = \omega$ then X is called *Lindelöf*.

An equivalent definition of the Lindelöf number: $l(X) = \min\{\tau :$ every open cover of X has an open refinement of cardinality $\leq \tau$ that covers $X\}$.

The *celullarity* (or *Suslin number*) of X is $c(X) = \min\{\tau :$ every pairwise disjoint family of nonempty open sets in X has cardinality $\leq \tau\}$. If $c(X) = \omega$ then X is called a *CCC space* (CCC = the Countable Chain Condition).

A family $\mathcal{B}$ of non empty open sets is a *π-base* of X if very non empty open set contains an element of $\mathcal{B}$. The *π-weight* of X is $\pi w(X) = \min\{\tau : X$ has a π-base of cardinality $\leq \tau\}$.

A *network* (in the sense of Arhangelskii) is a family $\mathcal{N}$ of subsets of X such that for every $x \in X$ and every neighborhood $U \ni x$ there is $N \in \mathcal{N}$ such that $x \in N \subset U$. The *network weight* of X is $nw(X) = \min\{\tau : X$ has a network of cardinality $\leq \tau\}$.

Continuous surjections are called *condensations*. The *i-weight* of X is $iw(X) = \min\{\tau : X$ can be condensed onto a space of weight $\leq \tau\}$. (Recall that we assume all spaces to be Tychonoff, otherwise we would have $iw(X) = \omega$ for every X.)

The *spread* of X is $s(X) = \sup\{\tau : X$ has a discrete subspace of cardinality $\tau\}$.

The *extent* of X is $e(X) = \sup\{\tau : X$ has a closed discrete subspace of cardinality $\tau\}$.

If φ is a cardinal function, then $h\varphi(X) = \sup\{\varphi(Y) : Y \subset (X)\}$. If $hl(X) = \omega$ ($hd(X) = \omega$) then X is called *hereditarily Lindelöf* (respectively, *hereditarily separable*.)

- $w(X) \geq hd(X) \geq d(X) \geq c(X)$
- $w(X) \geq \pi w(X) \geq d(X)$
- $w(X) \geq nw(X) \geq \max\{d(X), l(X)\}$
- $w(X) \geq hl(X) \geq l(X)$
- $\min\{hd(X), hl(X)\} \geq hc(X) \geq c(X)$
- $w(X) \geq iw(X)$
- $|X| \geq \max\{hd(X), hl(X), nw(X), iw(X)$
- None of the inequalities above (except for the delicate interrelationships between hl, hd, and hc) can be reversed. Give examples!
- $w(X) \leq 2^{d(X)}$
- $s(X) = hc(X)$.