0.1 Some more questions (the series of warming-up exercises continued)

Compact sets without isolated points are called *perfect*. A subset $X \subset \mathbb{R}$ is called *Bernstein* if for every perfect $K \subset \mathbb{R}$, both $K \cap X$ and $K \setminus X$ are non-empty. A set is *of first category* if it is a countable union of nowhere dense sets. A set is *of second category* if it is not of first category. A space is *Baire* if no non-empty open set is of first category. Continuous bijections are called *condensations*. If there is a condensation from X onto Y we say that X *condenses* onto Y.

- Show that if X is a $>$ 1-point compact zero-dimensional second countable space, then $X^\omega \cong 2^\omega$.[1]
- Show that if X is a non-compact zero-dimensional second countable completely metrizable space, then $X^\omega \cong \omega^\omega$.
- $2^\omega \times \omega^\omega \cong \omega^\omega$
- Is $\mathbb{Q}^\omega$ homeomorphic to either of 1^ω, 2^ω, ω^ω?
- (Noted by Ronnie Levy) Let B be a Bernstein set. Then B^ω is not homeomorphic to either of 1^ω, 2^ω, ω^ω, $\mathbb{Q}^\omega$. *Hint:* B^ω is neither compact nor completely metrizable nor of first category in itself (since the product of countably many second countable Baire spaces is Baire - the later is a result of Oxtoby found by Levy).
- Let B_1 and B_2 be two Bernstein sets. Must B_1 and B_2 be homeomorphic? *Hint:* There are $2^\mathfrak{c}$ Bernstein sets and only $\mathfrak{c}$ many continuous mappings from one separable metrizable space to another.
- Let B_1 and B_2 be two Bernstein sets. Must B_1^ω and B_2^ω be homeomorphic?
- Say that X and Y are *ω-equivalent* if $X^\omega \cong Y^\omega$. How many classes of ω-equivalence are there in the class of all zero-dimensional second countable spaces?
- Suppose X condenses onto Y and Y condenses onto X. Must X and Y be homeomorphic?
- ω condenses onto $\mathbb{Q}$.
- ω^ω condenses onto $\mathbb{Q}^\omega$.
- Can $\mathbb{Q}^\omega$ be condensed onto ω^ω?
- Can $\mathbb{Q}$ be condensed onto a compact space?
- Can $\mathbb{P}$ be condensed onto a compact space?
- How many pairwise disjoint 8-s one can draw in $\mathbb{R}^2$? ... in $\mathbb{R}^3$?

[1] $\cong$ means "is homeomorphic to".